The Geometry of Life

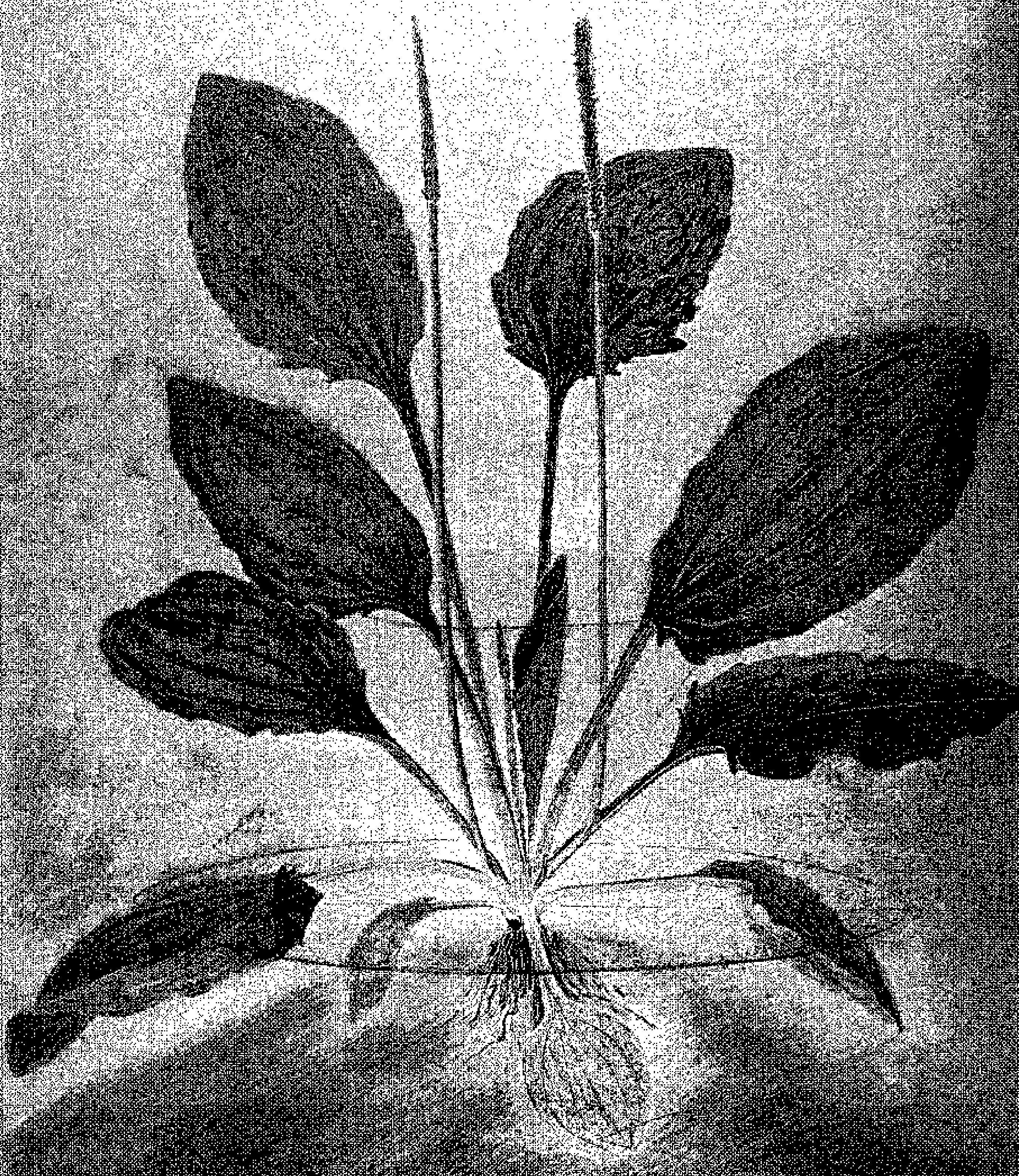
TOWARD A SCIENCE OF FORM

Amidst nature's everchanging raiment, we descry what may be her greatest miracle – her constancy. Each year, fresh, new soil, water, and light weave to form the familiar shapes of leaf and flower. The memory of last year's forms silently lives through a wintery night to unfold under warmer skies into the foliage of spring. Nature remembers the maple's leaf just as we do. The miracle deepens as we turn from the kingdom of plants to that of animals. For while leaf and stem die each fall into the earth, the life of beast and bird is not so completely bound to the seasons. They – and we – persist from season to season, year to year, even though every cell of our bodies is changing: perishing, to be recreated. Every seven years we are filled out anew. The familiar countenance is familiar not for its substance but for its lineaments, its shape or gesture. In it we recognize a form that passes through all change. As Heraclitus put it two thousand years ago, "It is in changing that things find repose." The world, in constant flux, rests.

Such considerations as these have stirred philosophic and scientific reflections since at least the time of ancient Greece. Plato raised form to the realm of incorruptible, eternal *eidos*. His pupil Aristotle paired form with matter and named them the twin principles that underlie all being and becoming. Throughout the history of science, the mystery of form's origins has been the central question. Why are things formed as they are: why does the pyrite crystal show itself as cube and dodecahedron only, why do leaves spiral around the stem, why is the heart shaped and structured as it is? Again and again one encounters among biologists the judgment expressed by Joseph Needham that "the central problem of biology is the form problem."

It is this aspect of nature, the aspect of form, that we shall explore here, in a brief introduction to a man whose discoveries open new avenues of inquiry, not only into the forms of nature, but into the very nature of form itself. His particular study, and ours for the purposes of this article, is that of mathematics, which is concerned with pure form, form

by Arthur G. Zajonc



From the essential polarity of spatial structure revealed in projective geometry ... is derived a scientific concept of formative forces specific to the living world, manifested in the phenomena of growth and form. ... The new conception gives a way of access to the essential wholeness of living things, complementing the one-sided approach which seeks the source of life in ever smaller particles.

From *The Plant Between Sun and Earth*

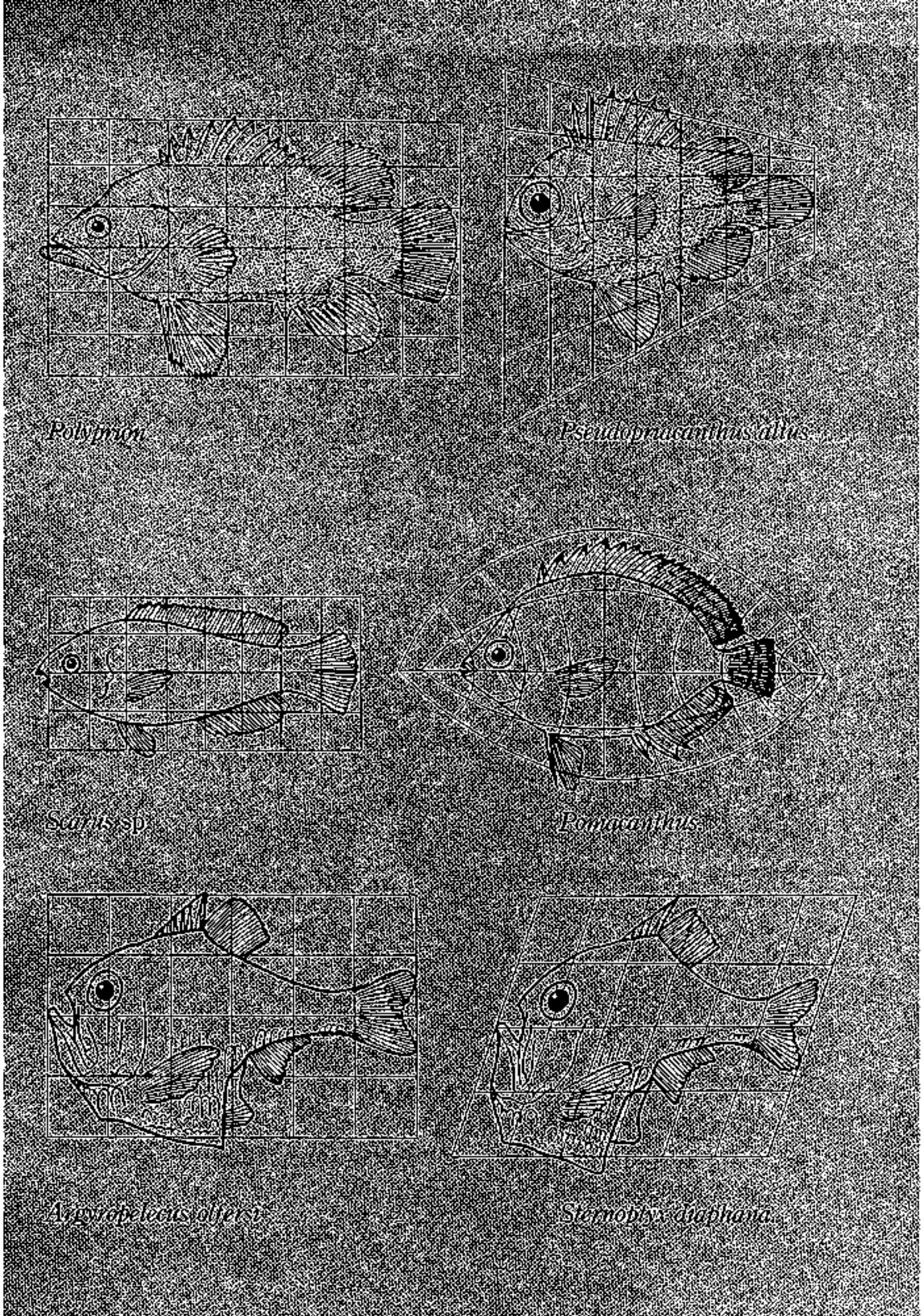


Figure 1. Transformations of the outline of the fish Polyprion *to yield the outline of another species,* Pseudopriacanthus altus, *according to the method of D'Arcy Thompson. Similar transformations are also illustrated*

winnowed completely from the corporeality that always accompanies it in the sense world.

Since ancient times, arithmetic and geometry have divided the universe of mathematics between them. The queen, however, was geometry, who reigned unchallenged from the time of Euclid until the eighteenth century. Only during the Enlightenment, at the hands of men such as Lagrange and Laplace, was geometry dethroned and purely algebraic, abstract analysis put in her place. Lagrange informs us with evident satisfaction in his *Mécanique analytique* that "no diagrams will be found in this work. The methods which I expound in it demand neither constructions nor geometrical or mechanical reasonings. . . ."

By contrast, we shall be drawn into that realm of mathematics that speaks directly and visually of form, namely geometry, for our ultimate goal will be to describe a visible and highly ordered domain of natural phenomena.

To date, the geometry of the living world has essentially defied our attempts to imitate it. Perhaps our approach has been too limited in scope or too inflexible. The concepts we bring to bear are often bound by our very conception of space and the movements we think possible within that space. To comprehend the geometry of forms in the living world, new concepts may be needed, ones intimately wedded to organic phenomena. The task of finding such concepts is not a simple one, but there are paths that beckon and individuals who have begun to explore them. The path we are going to pursue here is that of projective geometry. It embraces a vision of space far more dynamic, far more general, than the one implicit in Euclidean geometry. By giving up the rigid, restricted movements of Euclidean geometry, we gradually rise to a wonderful, fluid form of geometry, one with which we may hope to capture at least a small portion of those forms the living world displays before us. This is, after all, a world characterized by becoming, by development, by growth and decay. It seems somehow appropriate to approach the living world with a geometry of becoming.

The study of geometry and pattern in the inorganic world is an old and honorable pursuit. From the hexagonal forms of the snowflake to the intricate dance of the planets, both the static and dynamic patterns of the physical universe have been systematically described.

The forms of plants and animals have been more resistant to precise mathematical description. True, individuals such as the Scottish biologist D'Arcy Thompson have in their treatment of organic forms drawn upon geometry to establish connections between a great variety of plants and animals. The apparatus of geometry, however, is introduced only toward the end of D'Arcy Thompson's classic *On Growth and Form,* and only in an elementary way. In the famous last chapter, "On the Theory of Transformation or the Comparison of Related Forms," he lays systems of coordinate nets over various animals or skeletal members. By imagining these nets as subject to particular systematic distortions, the form of one species can be geometrically transformed into that of another with remarkable fidelity. The adjoining drawing shows what I mean *(figure 1).* As an example, he takes the fish *Polyprion* and places over it a rectangular coordinate system and then transforms it to an alternate coordinate system to yield the species *Pseudopriacanthus altus.* The original form, that of the *Polyprion,* is given rather than constructed, and thereafter is transformed point by point via his "method of transformed coordinates."

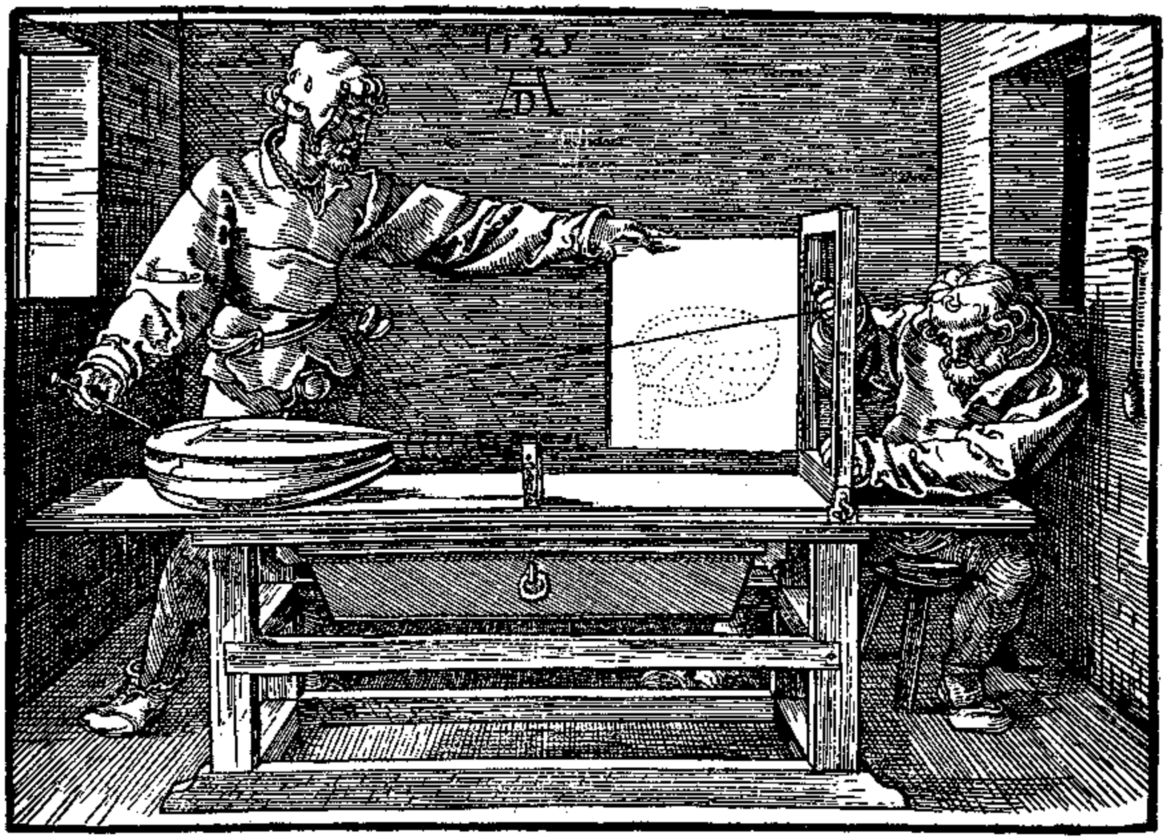


Figure 2. Woodcut by Albrecht Dürer illustrates the principle of projection and section. In the woodcut the artist marks the points at which the rays from the eye to the lute intersect the screen.

D'Arcy Thompson's work is only a hint at a more general and powerful use of geometry in the study of form in nature. As I hope to show, if we consciously develop geometry with the principles of transformation foremost, we gradually move from the most elementary to more and more complex transformations. In so doing we are following the program put forward in 1872 by the brilliant mathematician and pedagogue Felix Klein. His work, and especially that of his Norwegian collaborator, Sophus Lie, provide the basis for a geometry that can be used to study certain of nature's forms. This study has been pursued by several geometers, of whom Lawrence Edwards is the most recent and most successful. Much of this article concerns the discoveries he has made and continues to make. But before we enter into Edwards's study of organic forms, I would like to try to give the reader some sense for the mobility and beauty of the mathematical thought with which he works.

When asked by King Archelaos for an easier way into geometry, Euclid is said to have replied, "There is no royal road to geometry." The nineteenth-century German mathematician Hankel was certainly thinking of Euclid when he called projective geometry the royal road to all mathematics. Once one travels a way along that road, Morris Kline's more recent sentiment quickly becomes one's own: "In the house of mathematics there are many mansions, and the most elegant is projective geometry." Yet for all that it is a mostly forgotten mansion today, and so we must spend a moment or two retracing its elements.

It is to the Renaissance artists of the fifteenth and sixteenth centuries that we must turn for projective geometry's beginnings. The discovery of perspective brought about the extraordinary transition in painting from two to three dimensions. We need only compare the spatial arrangement and composition of medieval paintings by such artists as Giotto, Duccio, and Simone Martini to those of works by Dürer and Leonardo to realize that before 1500 space, size, and composition obeyed spiritual or symbolic laws, not the physical laws of perspective.

With the discovery of perspective is born the basis for projective geometry. Dürer's 1525 woodcut, "The Designer of the Lute," shows clearly the fundamental operation of projection and section so central to projective geometry *(figure 2)*. To assist us in understanding this construction, let us imagine that "visual rays" are emitted from the eye. The object before us, in this case the lute, is touched by each visual ray and the object is thereby perceived. Now between the eye and the lute place a screen. The rays from the eye to the lute are intercepted by the screen, forming a "perspective" view of the lute on the screen, as Dürer shows us. Here we have the key construction of projective geometry: projection from a center (the eye) and section by a plane (the screen). In the process we have "transformed" the object, that is, created an image by identifying one point on the screen with each point on the lute. This is the mathematical definition of a "point transformation." By tipping the screen or moving the center of projection, an enormous range of transformations becomes possible. We can also take the screen as a new object and transform it in the same way by a second transformation, placing a second screen between the first one and the center of projection. The business of projective geometry is to investigate the laws of the patterns that arise in space through a series of such transformations.

When confronted by the whirl of movement that projective transformations entail, it may seem difficult to imagine any stable ground or lawfulness. However, by making the transition slowly from the simple transformations associated with ordinary Euclidean geometry to the far more general ones of projective geometry, we can be led to experience an element of order that persists throughout.

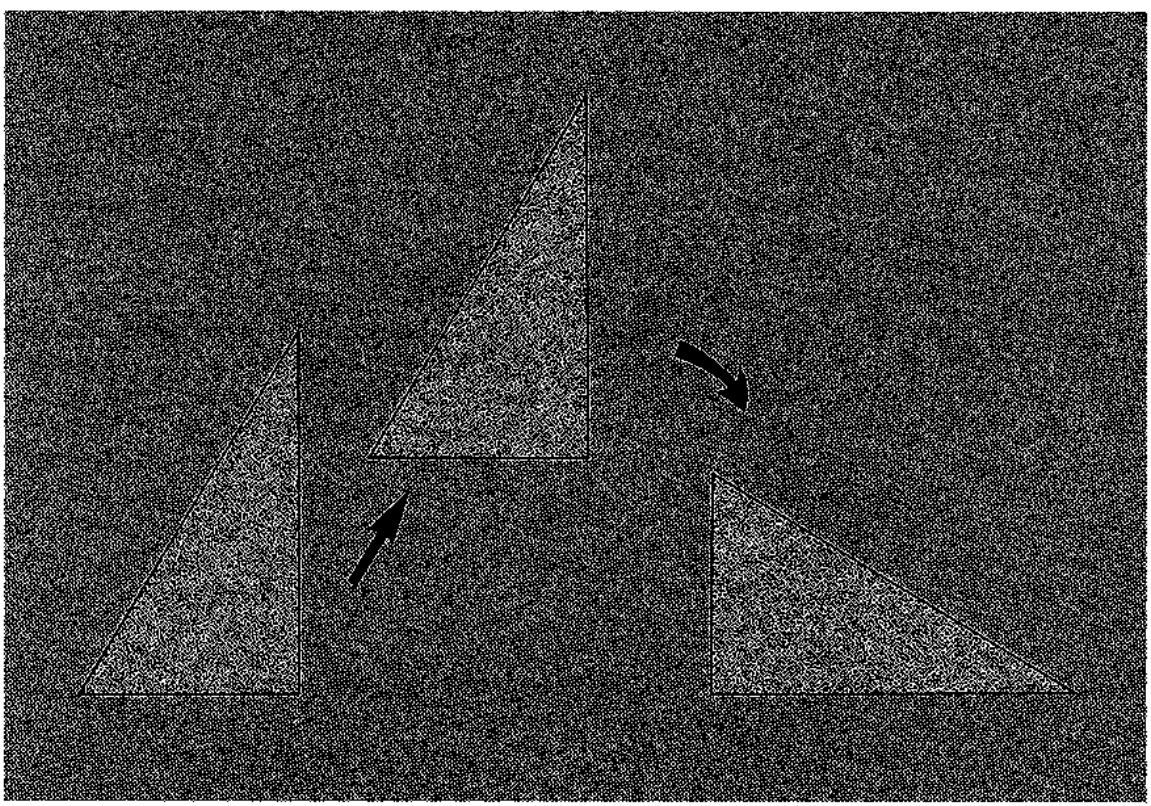
As the following discussion will show, a triangle projected onto a surface can assume many different triangular forms, depending on the angle of its projection. Yet certain properties will remain unchanged. The straight lines that compass the triangle, for instance, will always reappear as straight lines. Thus is "straightness" one of the invariants, or unchanging elements, of projective geometry. What other elements exist such that "in changing they find repose"? By pursuing this question we come not only to the stable ground of geometry, the laws of space that govern projective transformations, but also to a special set of forms, some of which will be startlingly familiar.

To learn how these special forms arise, we must first bring geometry into motion, for only against the backdrop of incessant change does the concept of repose, or "invariance," gain meaning. Imagine you have before you a triangular piece of paper. It can easily be slid about the tabletop to assume any number of positions. The accompanying figure shows three such positions *(figure 3).* To move from one to the other I can push the triangle up and to the right, and then rotate it. If I take my ruler, I find the corresponding sides of the triangles are still of the same lengths. My protractor likewise shows that the corresponding angles are of unchanged magnitudes. The triangles are, as Euclid would say, congruent. Neither length nor angle has changed in the process I have just described. Formally, one would say that lengths and angles are "invariant" under translations and rotations. Here we discover one kind of motion, by noticing that lengths and angles remain unchanged.

Yet clearly there are other kinds of motions or changes possible. The cubic form of a pyrite crystal is forever the same, yet it may grow in size. That is, the lengths of its sides will change, without an associated change in the angles. Here we encounter a new kind of transformation, one that involves a change in size but not in shape. Thus to translation and rotation we add dilation as a possible transformation.

We may proceed stepwise to ever freer types of transformations, ones that will take us beyond Euclidean geometry. With each step, what was before an invariant enters the realm of change. In the case of our original triangle, not only lengths, but angles too were invariant. We then allowed the lengths of

Figure 3. A triangle can be moved from one position to another by translation *(left arrow) and by* rotation *(right arrow). Such simple transformations fall within the bounds of elementary Euclidean geometry.*



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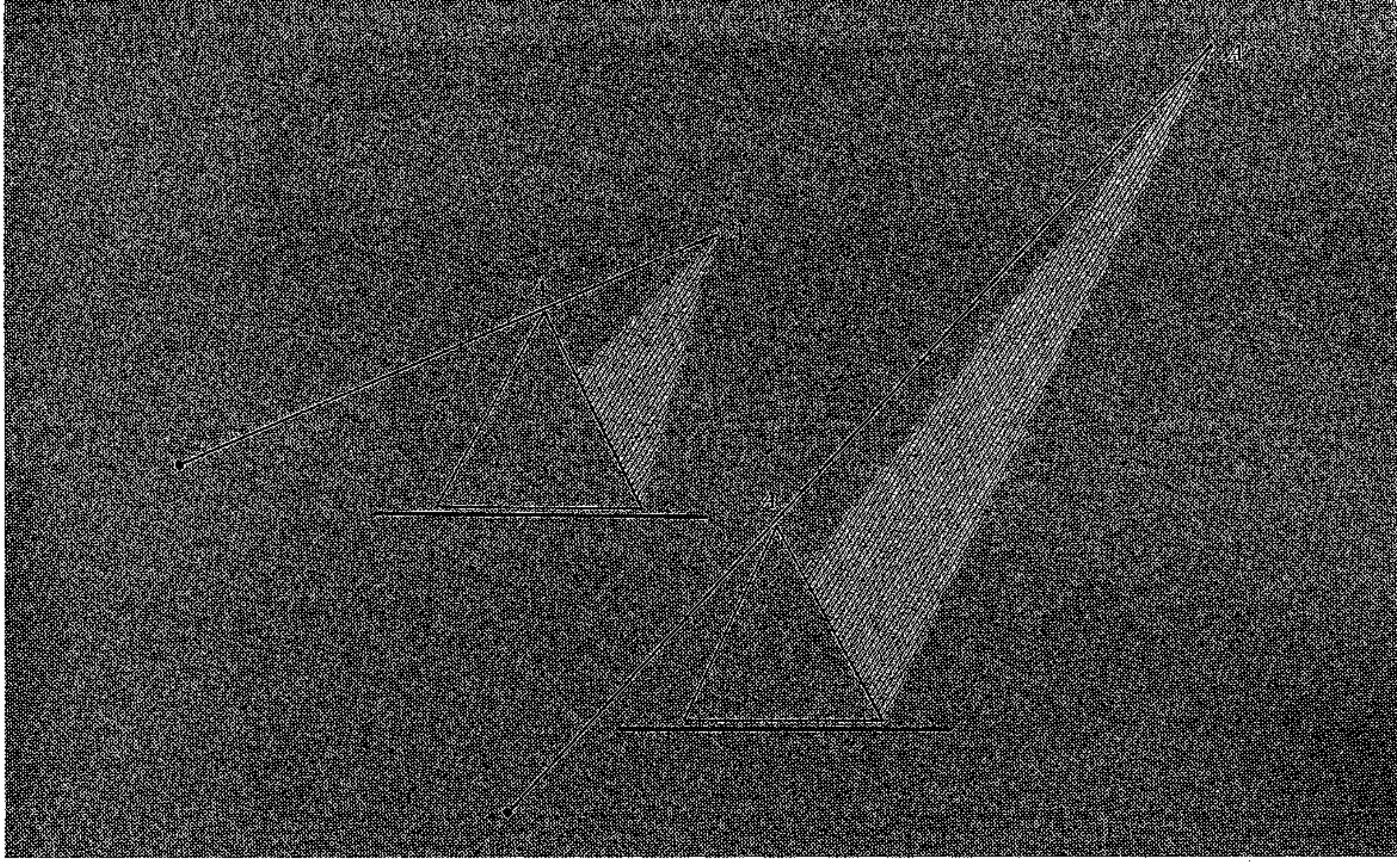


Figure 4. A triangle, illumined by a small bulb (the center of projection), casts a shadow onto a plane. The shadow represents a "projection" of the triangle; point A' in the shadow corresponds to A in the original triangle. As the center of projection is gradually lowered, the apex of the shadow (A') recedes and passes to infinity, and even returns from infinity in the opposite direction (not shown).

the sides to vary, yet only in such a way that all the angles remained the same. In other words, the sides all grew simultaneously. Now we will change angles as well as size, thus entering the realm of "affine geometry." We can do this by replacing our original triangle with one of rubber. Such deformations occur constantly in nature. Consider a stream of water. If you could enclose a portion of a brook in an imaginary flexible cube, then the cube, through which the brook flows, would tip and stretch, because the water nearer the brook bed moves more slowly. D'Arcy Thompson often uses such transformations as these in his *On Growth and Form.*

And so we may continue our pursuit of ever more flexible transformations. Though not obvious, certain invariants remain, even in this last class of transformations. It is rather remarkable, for instance, that under these transformations a set of parallel lines is transformed into another set of parallel lines – all the more astonishing when we remember that the angles between intersecting lines may change in general. One may state the invariance in another way. In plane geometry, two lines intersect at one point, unless the lines are parallel. We may overcome the exceptional character of parallel lines by defining a new "ideal" point, namely the point at infinity. Since under affine transformations, a set of parallel lines remains parallel, then the point at infinity remains a point at infinity. In projective geometry, even this invariant disappears. Infinitely distant elements can be brought into the finite by a projective transformation. We can easily see how this occurs in the next set of figures.

Imagine our ever-ready triangle as standing upright on a plane surface *(figure 4).* A small light bulb illumines the triangle, casting a shadow onto the plane. The shadow we call a "projection" of the triangle onto the plane. The apex A of the triangle is projected to A'. But notice what happens if the light is lowered. The apex of the shadow triangle recedes farther and farther, until it vanishes into the infinite horizon. The finite has become infinite. In projective geometry, we replace the light bulb with its mathematical analog, a center of projection. By lowering the center of projection, the apex can be made not only to recede to infinity, but even to return again from the other side! It is as if passing to infinity in one direction brought one back from the opposite. Such is the nature of a projective transformation. With it we attain a very high order of freedom, yet even here there are properties and forms that remain unchanged. "Straightness" is one of them. Another is the property of "incidence." That is, if two lines intersect in a point before transformation, they

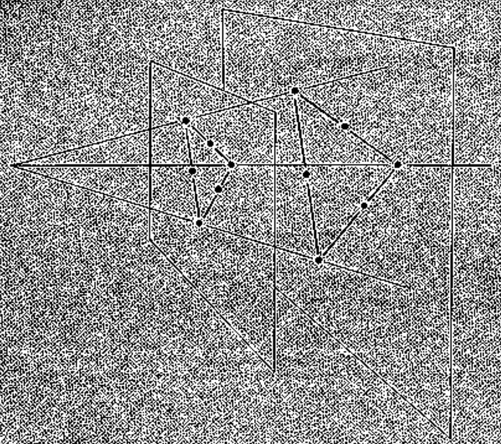
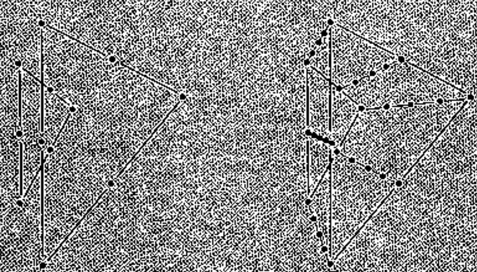


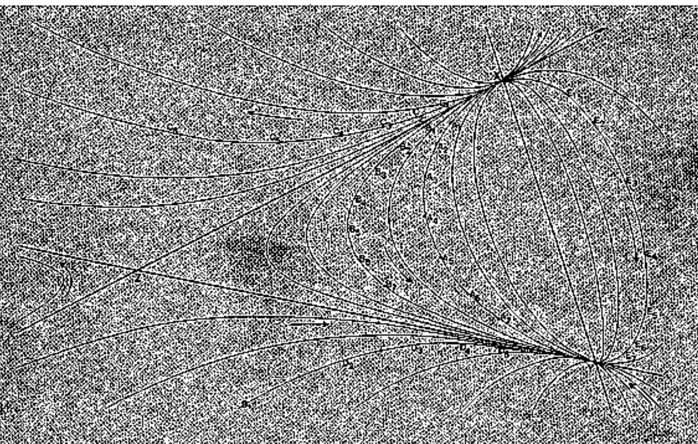
Figure 5

Projection of a small triangle (the object) onto a plane produces a second larger triangle (the image), thus illustrating the fundamental operations of projective geometry, projection and section.



If the planes are now united (or merged), the two triangles come to lie in one plane, as shown.

Imagine that the projective transformation shown above has been repeated several times and that five separate planes have been united. Five triangles now lie in one plane. Corresponding points in the triangles can be connected into a smooth line termed a path curve. *In the example shown here the path curves appear as short straight lines but under different circumstances of projection would be seen as curves. The three invariant points of the transformation are not illustrated.*



Projective transformation of points in a plane can be performed in an entirely different manner to produce the path curves shown here. The points A_1, A_2, A_3 . . . represent successive transformations along a particular path curve. The direction of movement is indicated in the diagram with arrows. In this example the three points of the triangle are invariant and do not move under repeated applications of the transformation.

will intersect in a corresponding point after transformation. There are still other invariants, but for our purposes we can limit our treatment and turn now to Sophus Lie, in whose work the idea of invariance meets with that of form in space, the heart of our considerations.

Nearly one hundred years ago Lie presented a systematic discussion of a special set of curves in two dimensions, which he termed *Bahncurven* or "path curves." These curves, and the analogous surfaces in three dimensions, possess the remarkable property of remaining unchanged when acted upon by repeated application of a projective transformation.

Path curves can be seen as arising in the following way. Recall our discussion of Dürer's drawing of the lute. I commented there that we could take the screen as a new object and project it onto another plane or screen by means of a second projective transformation. Clearly there is no end to the number of times such an operation could be repeated, the new screen now becoming the object.

Now imagine three lines forming a triangle drawn onto a thin glass plate; several points around the perimeter of the triangle are carefully marked with blue dots *(figure 5)*. Some distance above the glass plate is a small lamp, our center of projection. We imagine the shadow to fall on a second, cleverly fabricated glass plate, which turns black exactly where the shadow falls and produces blue dots at the proper corresponding points. We have just performed the fundamental transformation of projective geometry, projection and section. (Whereas in the Dürer drawing, the plane is between the object and the center of projection, in this instance the object is between the plane and the center of projection.) Placing the two plates together and looking through them, we see two triangles, one slightly different from the other – the degree of difference depending on the particulars of the projection and section. If the second plate was very close to the first, then the difference can be small indeed. Thus one has two triangles in one plane. Mathematicians formalize the process by saying that the first plane and the second are united after the transformation.

The process can be repeated with the object plate, image plate, and projecting lamp all situated exactly as before. The image plate is now projected. After the planes are united, three triangles appear, each with its set of blue dots. By repeating the process over and over, many triangles appear, all with blue dots. The mathematician would say that we have transformed the plane onto itself many times via a series of identical projective transformations. Now forget the triangles and attend only to the blue dots. They will form a set of curves. Following the trajectory of one of the dots, a point in a plane, we have been led to a path curve. Although we have chosen to watch only a few blue dots, clearly all the points of the plane are brought into movement by the series of projective transformations. Transforming the plane onto itself has produced path curves. They are forms of the plane, structures that remain unchanged throughout the movement. By changing the angle of projection, we could arrive at a different series of projective transformations and a different set of path curves.

If we considered the entire plane, we would find that three and only three points never move at all. They are completely invariant.* All other points move along path curves that cross only at the three invariant points of the plane. We can watch the points of a path curve march dutifully along behind one another, never deviating from their designated path, as if moving through the veins of some organism. The whole plane is in movement. Yet within the flux, there abides form: the pattern of path curves does *not* evolve, although every aspect and point of the plane (save three) are in motion! One cannot help noticing the kinship between such form within movement in geometry and the similar biological phenomenon. Every human cell is replaced within a seven-year span, yet our countenance remains, in all essentials, unchanged. The beauty apparent in the contemplation of these concepts and forms quickly wins the heart of anyone with the least affection for the elegance of pure mathematics.

While far more difficult to imagine, an entirely analogous procedure can be followed in three dimensions. In this case, surfaces as well as curves fill out space with forms. These are the invariant forms of space; invariant, that is, under repeated applications of an identical projective transformation. It is just these dynamic yet invariant path curve forms that we shall discover around us in the plant and animal kingdoms.

*The basis of this statement can be shown mathematically but is too complex to present within the confines of this article.

Path curves present a rich variety of spatial forms. These include egg shapes, cones, and vortices *(figure 6)*. Using a particular mathematical procedure, one can assign a number, called lambda, the Greek letter λ, to each shape that appears. For instance, positive values between zero and infinity are associated with various egg-shaped path curves. Negative values give all the vortex forms. The forms so created on the geometer's drawing table bear a striking resemblance to certain forms in nature. Lawrence Edwards starts with the question, is this resemblance merely superficial, or does a genuine correspondence exist?

During twenty years of research, Edwards has explored the kinship between path curves and natural forms as diverse as pine cones, plant buds, eggs, the human heart, and developing embryos. The results of his research and his reflection on their meaning are summed up in his recent book, *The Field of Form*. In it he tells of the blind alleys into which he wandered but also of the moments of excitement when he saw clearly how transformations of projective geometry touch the earth and gather up substance to clothe their forms. We will inquire into only two of his findings, those concerning plant buds and the human heart. With them the beauty of his work will become apparent.

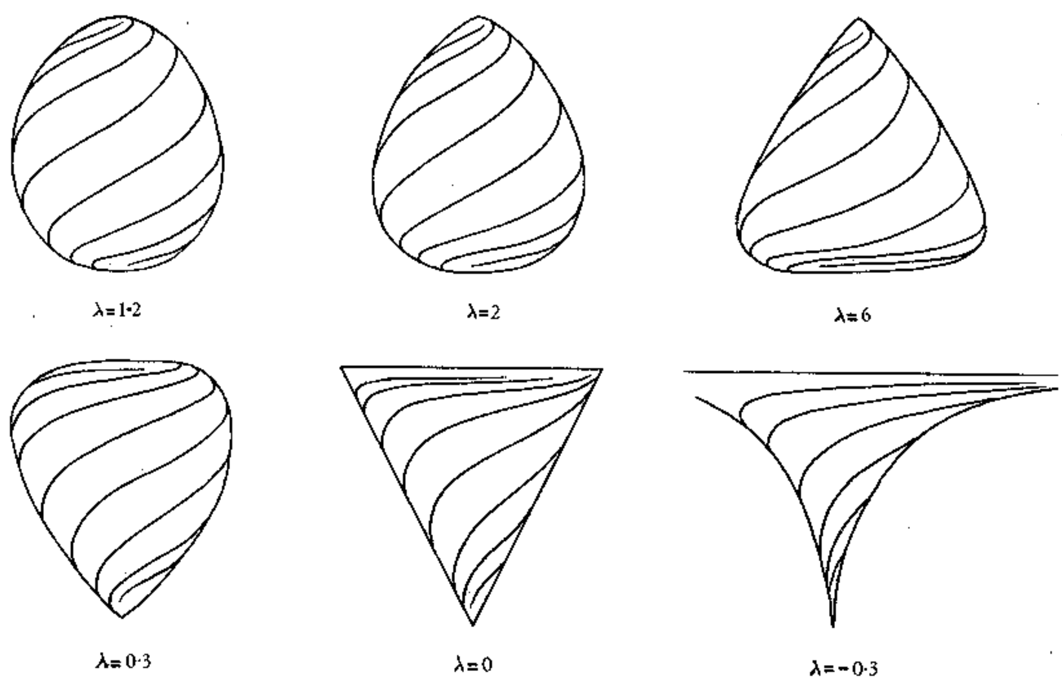


Figure 6. Path curve forms in three dimensions resemble egglike and vortexlike figures, depending upon the manner of their construction. The several examples shown here represent path curve forms differing only in the important mathematical parameter, lambda (λ). If lambda = 1.2, the form is rounded and egglike. As lambda increases, the form becomes sharper and blunter at its ends. Negative values of lambda produce vortex forms.

Let us begin with the bud of the wood sorrel *(Oxalis acetosella)*, which gives forth its small white flower during midsummer. By carefully collecting several samples, mounting them for photography, and enlarging the prints, we can make very exact measurements of the bud shape in profile. In constructing the corresponding path curve, we find that by placing two of the invariant points at the upper and lower poles of the bud and following an exact mathematical procedure, we can determine the path curve that best fits the wood sorrel bud. This can be done for many, many wood sorrel buds at the same stage of development, regardless of size. The agreement between the pure mathematical form and the living one is striking. Perhaps the most immediately convincing evidence is found in the simple visual comparison of an ideal path curve with the actual form of the bud. Often the difference is little more than the width of a pencil line—well within the precision with which one can make reliable measurements on these frail little buds. Not all species follow path curve forms so perfectly, but in over eighty percent of the cases studied, the plant buds are found to reflect path curve geometry with remarkable fidelity.

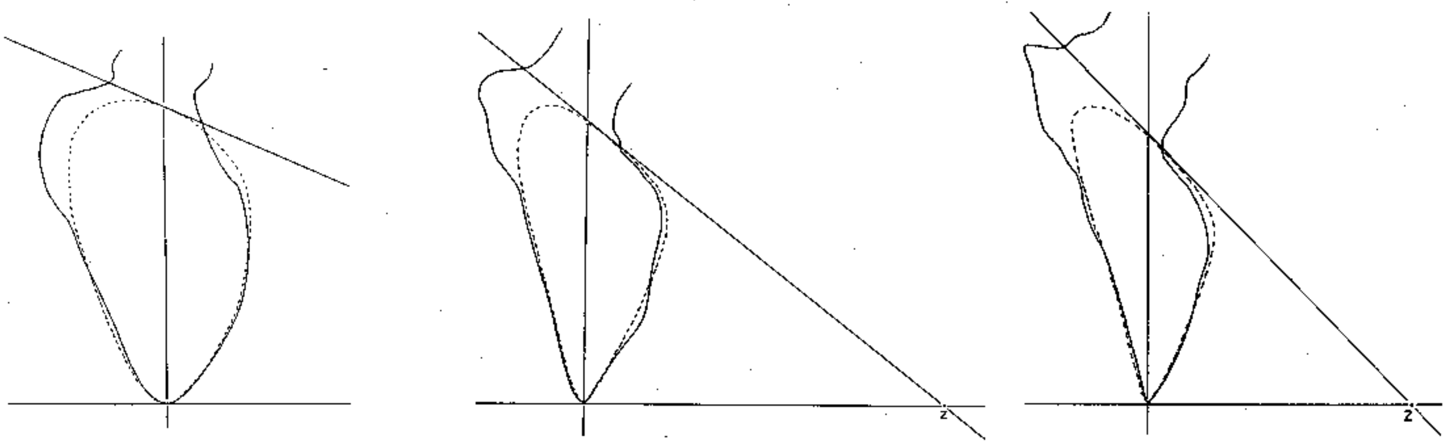


Figure 7. The solid lines represent tracings of the actual living form of the ventricle as detected by an X-ray procedure. The broken lines illustrate the path-curve forms calculated to fit the actual forms. During one full heart cycle (of about 0.8 seconds' duration), the path-curve forms change dramatically and reveal a rhythmic seven-fold process.

The left ventricle of the human heart at the moment of full diastole or relaxation (left).

The same ventricle 0.14 seconds before systole or contraction (middle).

The ventricle 0.04 seconds before full systole (right).

Correspondences can be found elsewhere in the living world. The spiral tendency of leaves on a stem has long engaged botanical and mathematical researchers. Similar spiral configurations are to be seen in pine cones and in bud formations, in the way petals arrange themselves around the bud center. It turns out that the path curve surfaces of the bud are themselves covered with a spiral pattern, each spiral being a path curve. Very often one can capture the gesture of these spiral patterns by suitable path-curve analysis. Such agreement seems unlikely to occur by chance, for it can be found in many other biological forms, including that of the organ of the heart.

The heart in animal or man can be thought of as the perfect or archetypal muscle. Other muscles may be seen as variations of this central organ, whose whole existence is ceaseless rhythmic activity. Beginning with the detailed studies of the heart made by Scottish anatomist J. Bell Pettigrew in his book *Design in Nature*, Lawrence Edwards worked to uncover the path-curve form of the heart. In this instance, not only was the outer form of the heart significant, but so were the particular circling patterns made by the several layers of muscle that together comprise the heart. Pettigrew distinguished seven layers. Moving from the outermost inward, the muscle patterns change from a left-handed to a right-handed spiral at the fourth layer. In addition, the left ventricle, which Pettigrew terms the "heart of the heart," changes its form as one moves from layer to layer. Would it prove possible to follow these changing forms as one moved inward? Indeed, by changing the positions of the invariant points, the slightly asymmetric form of the heart can be geometrically reproduced. The form of the left ventricle also proved possible of capture in a path curve. Even the spiral gesture of the muscle layers, like the spiral of the wood sorrel bud, finds its expression in path curves, as a second set of curves that cover the surface.

One of Edwards's most dramatic accomplishments must surely be his study of the living human heart, made possible through a kind of X-ray moving picture. Every fiftieth of a second an X-ray image is taken of the beating heart. The technique provides a picture of the inside surface of the heart, the innermost of Pettigrew's seven layers *(figure 7)*. Edwards follows the changing form of the heart throughout the duration of a pulse and finds that the movement from full expansion to full contraction is in itself a rhythmic sevenfold process, one beautifully revealed through his path-curve analysis.

So far we have seen a remarkable congruence between those forms inwardly created by the human mind, that is, path curves, and the tangible forms of plant bud and heart. Lawrence Edwards has made preliminary studies in several other directions, but we must leave these aside for want of space. Of much greater importance is his discovery of what he terms the "pivot transformation," which relates forms in space to those of a complementary realm, one that is sometimes called counterspace. I shall conclude by spending a few moments considering the general character and significance of these ideas for the understanding of plant forms.

The forms... created on the geometer's drawing table bear a striking resemblance to certain forms in nature. Lawrence Edwards starts with the question, is this resemblance merely superficial, or does a genuine correspondence exist?

When visualizing a circle, we tend to see it as a continuous curve formed of points all equally distant from the circle's center. The circle is formed from the center out, point by point *(figure 8)*. It is initially surprising to learn that there is a second means of forming a circle. We must free ourselves from the habit of thinking of points as somehow more primary than the line. For in this other view, it is just the line, and not the point, that is used to generate a circle. The construction can easily be understood by visualizing one line after the next as touching a circle. The set of lines has thereby created a tangent envelope that also completely defines the circle *(figure 9)*. If we generalize still further to three dimensions, the infinitely extensive, unitary, and undivided plane becomes the generative entity of space. Thus is the sphere formed no longer of points equi-

Figure 8. Pointwise circle.

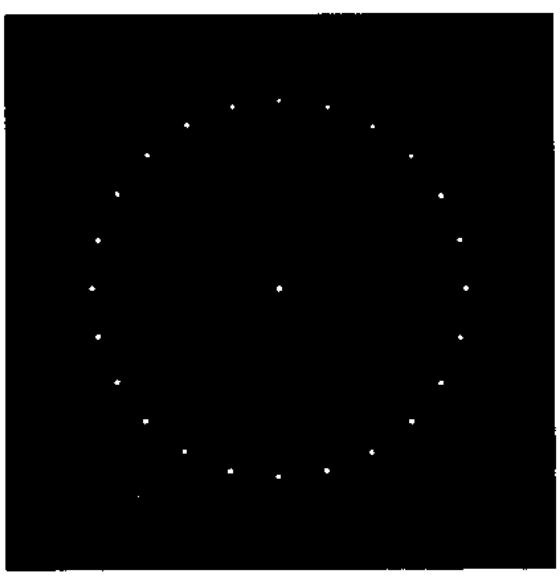
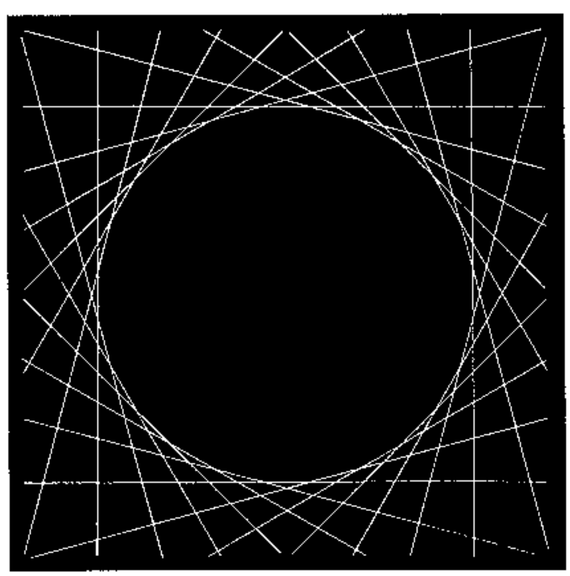


Figure 9. Linewise circle.



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Wild rose bud with immature seed capsule.

distant from a given point. Rather, planes shape the sphere, just as the sculptor shapes his clay with the flat of his hand. So may the infinitely many planes of space fashion geometric forms from the periphery inward. It becomes possible to imagine a new kind of space, a "counterspace," wherein point becomes plane.

Working from indications for projective geometry given by the Austrian philosopher and scientist Rudolf Steiner, George Adams and Louis Locher-Ernst sought to develop a geometry of counterspace and to connect it with the botanical kingdom. (Adams's work is to be found in his books *The Plant between Sun and Earth* and *Physical and Ethereal Spaces,* both coauthored with Olive Whicher.) Lawrence Edwards, who had worked with Adams, has continued these efforts. In particular, he has explored a novel class of projective transformations that involve a change in space element. That is, instead of transforming one point to another, or a line to a line, Edwards uses those projective transformations that transform a point to a line, or a point to a plane.

Such transformations immediately call to mind what the quantum physicist David Bohm terms "the implicate order," wherein the entirety of a line can be "enfolded" into a point. In such instances the relationship between the whole and the part is clearly unusual, for the whole is in the part, the line is in the point!

We cannot delve here into the complexities of counterspatial geometry. Suffice it to say that once we have explored its properties mathematically, we are free to move between space and counterspace, between point and line, by means of Edwards's pivot transformation. Can this possibility be exploited in the study of organic forms? Lawrence Edwards saw the means for doing so. Working with the hip of the wild rose, he was able to discover the beautiful "plane-wise" vortex that stands in counterspace behind it. Moreover, the character of the pivot transformation is such that the bud of the wild rose (itself a path curve) mediates the transformation from vortex to hip *(figure 10).* Thus are all three elements—bud, hip, and vortex—brought into an harmonious interrelationship. Other plant species showed similar fidelity to the geometric forms generated from his counterspatial vortex.

The discovery of the counterspatial vortex, as he describes it in his book, is a grand moment to rehearse with Mr. Edwards. Through it he seems to approach the nature of life itself. And now the full

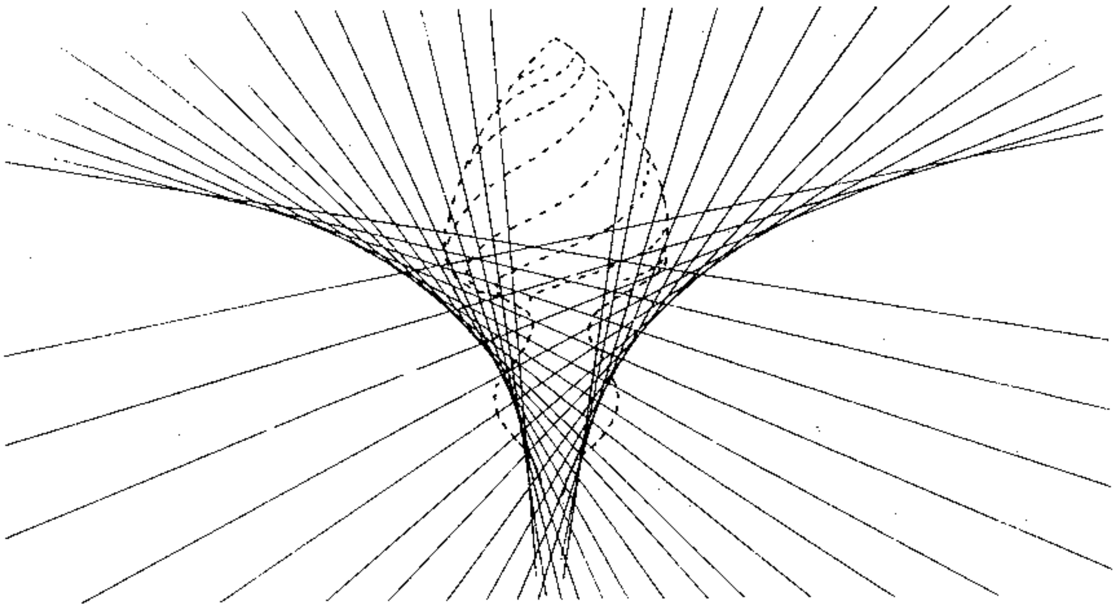

Figure 10. Through use of the pivot transformation, it can be shown that a path-curve vortex form mediates between the form of the bud and the form of the seed capsule, in this instance, a rose hip. The seed-bearing organ of the plant is usually not a path curve, but its form can be delineated with great accuracy by the pivot transformation.

strength of projective geometry becomes clear. In addition to providing transformations that are highly mobile, it establishes, through the development of counterspace, a new relationship between the whole and the part.

In his work, Edwards not only is concerned with describing mathematically the natural forms he studies, but also tries to find the origins of these forms. He has not, as do most contemporary researchers, sought to find them through molecular biology, but instead by developing a science of form. When reading about what he terms "fields of form," one is reminded of the great English physicist Michael Faraday, developer of the field concept. The fields in Edwards's work, however, are conceived not as physical forces but as insensible, ideal forms that are nevertheless imaged in the tangible shapes of the living world. He is convinced, as was Goethe, that nature creates her infinite forms according to a plan, according to an Idea. Goethe wrote: "The Idea is eternal and unitary. . . . All that of which we become aware and of which we can speak are only manifestations of the Idea." The Idea is not to be identified with a purely material or molecular basis – the building blocks of life. Rather, we should attend to the forms themselves. In writing of biology, Aristotle made use of an analogy, that of a house:

> *The object of architecture is not bricks, mortar, or timber, but the house; and so the principal object of natural philosophy is not the material elements, but their composition, and the totality of form, independently of which they have no existence.*

Lawrence Edwards has attended to the composition and form of organic nature as few before him and has shown that through careful observation of nature and the free activity of human thinking, the Ideas that seem to touch nature may also unfold in the human mind.

When Kepler brought forth the great laws of planetary motion, he said he had stolen the golden vessels of Egypt. Kepler heard through these geometric laws the harmony of the spheres, and his decades of labor were requited. Lawrence Edwards shares Kepler's vision of the world as created and formed according to an image, fashioned not simply by a field of forces, but rather in accord with a "field of form." ●

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Suggested reading:

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